MATH 330 DO-AT-HOME MIDTERM
SPRING TERM 2008

INSTRUCTIONS:

- This is an *open book and notes* test, so feel free to use your textbook and class notes. However, you must work on your own. Please also do not spend hours in the library (or on the Internet) looking for solutions to these problems.
- If you get completely stuck on a problem, feel free to contact me, however, and I could possibly consider a hint appropriate.
- Exposition is important: please be as clear as you can about what you are doing.
- This is **due in class on Monday, February 18th**.

PROBLEMS:

1. In the context of Euclidean geometry (please specify exactly which axioms you are using, and quote/cite them thoroughly when you do use them):
   a) **Formulate a careful definition** of what it means for a point P to be "on one side" of a line ℓ. We wish particularly to know what it means for two points to be "on the same side" of a particular line. It would be nice also to **prove that**, given a line ℓ, anyy point P is either "on one side" or "the other" or on ℓ itself.
   b) **Formulate and prove** a statement to the effect that two circles always intersect in two points, one on each side of the line joining their centers. (Note: that statement is not quite true, part of the goal here is to state something carefully which is precisely true.)

2. **Make a picture** with `Geometry Explorer` which involves a construction of the *geometric mean* or *rectangle*. Don't be shy – the more sophisticated, the better: *e.g.,* make a spiral of golden rectangles, implement a `Geometry Explorer` "program" which builds a golden rectangle on top of a specified line segment or makes a second second near a given first one whose length is in the golden proportion to the first one, or... go wild! (The more elaborate, the more points!) Please e-mail me (at `jonathan.poritz@gmail.com`, as usual) your work as an attachment, either saved in an image format or in the `Geometry Explorer` file format (or both).

3. Using `Geometry Explorer`, draw a triangle and mark the following 6 points:
   - the midpoints of the three sides (call them *M1, M2* and *M3*), and
   - the feet of the the altitudes (call them *A1, A2* and *A3*; an **altitude** of a triangle is a perpendicular to a side which passes through the other vertex, and the point where this perpendicular intersects the side is called the **foot**).

   Find the **orthocenter** (the intersection of the three altitudes of the triangle) and then mark the points
   - the midpoints of the segments joining the vertices to the orthocenter (call them *m1, m2* and *m3*.

   Pick any three of the points you just constructed (the *M1, M2, M3, A1, A2, A3, m1, m2* and *m3*) and draw the circle passing through your three points.
   a) First, **prove** that the orthocenter is well-defined: that it is a single point!
   b) **Describe** what you notice about the circle you drew. **Explain** why the circle is called "the nine-point circle".
   c) **Prove** that the triangle with vertices *m1, m2, m3* is similar to the triangle you started with (**similar triangles** are the subject of §2.5 in the textbook) and the corresponding sides are parallel.

   Print a copy of your construction and include it with the work you hand in.

4. **Prove** that if a quadrilateral is inscribed in a circle, then the opposite angles are supplementary. (*Hint: this is* **Corollary 2.34** *in the textbook; its proof is* **Exercise 2.6.2**.)