NAME________________________________ 16 October 2008

MATH 224 — WORKSHEET/HOMEWORK

Do as many of the following as you are able during class. Please work in groups, and feel free to get help from Dr. Barnett. As always, fully explain your work (also the examples you give must be explained, as to why they have the desired properties). Finish tonight the problems you have not yet completed during class, and hand this sheet in as homework on Friday.

1) Use the Integral Test to show that $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ diverges.

2) What does the Integral Test say about $\sum_{n=1}^{\infty} n \sin(\pi n)$?

3) Write out the first five terms of $\sum_{n=1}^{\infty} n \sin(\pi n)$; does the series converge or diverge?

4) Is there any conflict between your answers to **2)** and **3)**? If so, how do you reconcile them?

5) The **Integral Test** states that if $f(x)$ is positive, continuous, and decreasing on $[1, \infty)$ and $a_n = f(n)$, then: $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ converges.

With the same $f(x)$ and a_n, is it true that $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_{17}^{\infty} f(x)\,dx$ converges?

What about "$\sum_{n=17}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ converges"?

For both, explain why or why not (the best explanation of "why not" would be an example!).

6) Is it possible to have a sequence all of whose terms are greater than 17 which converges? If so, give an example; if not, why not?

7) Is it possible to have a series all of whose terms are greater than 17 which converges? If so, give an example; if not, why not?

8) Give an example of a series which has at least 100 billion terms greater than 17 but which is convergent.

9) Give an example of a series whose terms are positive and tending to 0 and which is divergent.

10) Give an example of a series whose terms are positive and tending to 0 and which is convergent.

11) Give an example of a series whose terms are positive and all smaller than the terms in your example for number **9)** which is still divergent.

12) Give another example of a divergent series whose terms are positive and tending to 0, which is not merely a constant multiple of a series you have used above.

13) It is true that $\int_1^\infty \frac{dx}{x^2} = 1$ (check this!). From this, we can conclude that

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. **(B)** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = 1$. **(C)** Both **A** and **B**. **(D)** Neither **A** nor **B**.

As always, why?

14) What are the areas of the shaded regions?

(A)

(B)